MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

BMS1024 – MANAGERIAL STATISTICS

(All sections / Groups)

15 OCTOBER 2019
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FOURTEEN (14)** printed pages with:
   Section A: Ten (10) multiple choice questions (20%)
   Section B: Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restrictions.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.

1. The data below represent the amount of grams of sugar per serving of milk tea:

11	15	23	29	19	22	21	20	15	25	17

Determine the third-quartile for this data:

A. 23 grams
B. 20 grams
C. 19.73 grams
D. 15 grams

2. When a histogram is negatively skewed, it _____.

A. is symmetrical
B. tails off to the left.
C. has a slope smaller than one
D. tails off to the right

3. On average, a worker earns \$15 per hour at a plant in Australia and is told only 2.5% of all workers make a higher wage. If the wage is normally distributed with standard deviation of wage rates is \$5 per hour, find the hourly wage for the plant.

A. \$15 per hour
B. \$24.8 per hour
C. \$5.2 per hour
D. \$2.34 per hour

4. Based on a sample of data, the calculated mean is 67, median is 60.31 and standard deviation is 7.31. Find the Pearson's Coefficient of Skewness of this data sample.

A. -2.75
B. 0
C. $+2.75$
D. $+0.92$

Continued...

5. There are currently seven statistics classes being taught this semester at Wilmington College. The following data represent the number of students in each class:

20	25	18	32	22	30	28

Calculate the variance of this given data:

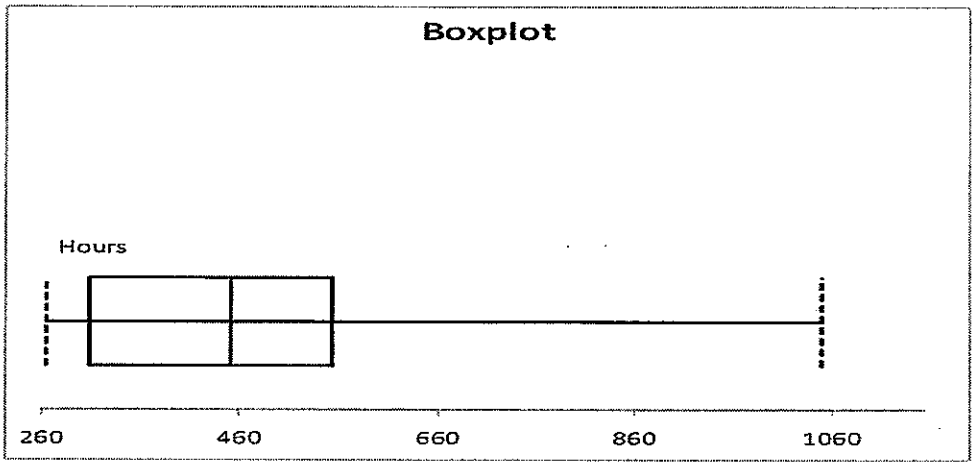
A. 27.67
B. 5.26
C. 23.71
D. 4.87

6. Health care issues are receiving much attention in both academic and political arenas. A sociologist recently conducted a survey of citizens over 65 years of age whose net worth is too high to qualify for Medicaid. Calculate the mean age of the senior citizens as following:

66.65 71.35 82.30 69.56 70.56 75.40

A. 66.66 years of age
B. 72.64 years of age
C. 87.16 years of age
D. 69.80 years of age

7. A manufacturer of flashlight batteries took a sample of 130 batteries from a day's production and used them continuously until they were drained. The number of hours until failure were recorded. Given below is the boxplot of the number of hours it took to drain each of the 130 batteries:



Continued...

Based on the boxplot, state the minimum number of hours it took to drain each of the 130 batteries.

A. 260 hours
B. 460 hours
C. 1060 hours
D. 660 hours

8. The following data shows the number of home runs hit by the top 12 home run hitters in Major League Baseball during the 2011 season:

32	33	33	34	36	37	37	38	39	39	41	43

Determine the inter-quartile range of this data.

A. 3.25
B. 9.75
C. 6.5
D. 6

9. Which of the following statements is **false** concerning the attributes of *z*-values?

A. *z*-values are positive for data values above the mean of the distribution.
B. *z*-values are negative for data values below the mean of the distribution.
C. *z*-values can be positive or negative for data values above the mean of the distribution.
D. z-values are equal to zero for data values equal to the mean of the distribution.

10. The average mathematics SAT score for incoming freshman at a particular college is 535 with a standard deviation of 60. The coefficient of variation for SAT scores at this college is

A. 11.21%
B. 13.97%
C. 16.65%
D. 89.17%

Continued...

SECTION B: STRUCTURED QUESTIONS (80 MARKS)

There are THREE questions in this section. Candidates MUST answer ALL THREE questions.

Question 1 (25 Marks)

(a) Most dial-up Internet service providers (ISPs) attempt to provide a large enough service so that customers seldom encounter a busy signal. Suppose that the customers of one ISP encounter busy signals 18% of the time. During the week, a customer of this ISP called 20 times,

(i) what is the probability between five until seven calls that this customer encounters the busy signals?

(4 marks)

(ii) determine the probability at least three calls that this customer encounters the busy signals.

(4 marks)

(iii) calculate the expected number of calls which encounter the busy signals.

(3 marks)

(iv) find the standard deviation of this distribution.

(3 marks)

(b) We Care Air needs to make a decision about Flight 105. There are currently three seats reserved for last minute customers. Last minute customers must pay \$475 per seat. The decision must be made as soon as possible whether to release the seats. The number of last minute customers requesting seats, X has the following probability distribution:

Number of last minute customers requesting seats	0	1	2	3
Probability	0.45	0.30	0.15	0.10

(i) Calculate the probability from one to three number of last minute customers requesting seats.

(3 marks)

Continued...

(ii) Compute the expected profit gained from last minute customers requesting seats.

(3 marks)

(iii) Determine the variance and standard deviation for the above random variable.

(5 marks)

Question 2 (25 Marks)

(a) Transmission Fix-it stores recorded the number of service tickets submitted by each of its 20 stores. The record was based from January 2017. The listed data as follows:

823	648	321	634	752	669	427	555	904	586
722	360	468	847	641	217	588	349	308	766

(i) Determine 90% confidence interval of the mean number of service tickets submitted in January 2017.

(7 marks)

(ii) The company management will award a financial bonus to any store manager who obtained less than 475 service tickets. Is there enough evidence at the 10% significance level to conclude that most of the store managers will receive a financial bonus from the company management?

(8 marks)

(b) A university bookstore is facing significant competition from off-campus bookstores and they are considering to target a specific class in order to retain student business. The bookstore randomly sampled 150 freshmen and 175 sophomores. They found that 46 percent of the freshmen and 40 percent of the sophomores purchase all of their textbooks at the university bookstore.

At the 5 percent significance level, is there a significant difference in the proportions of freshmen and sophomores who purchase textbooks entirely at the university bookstore?

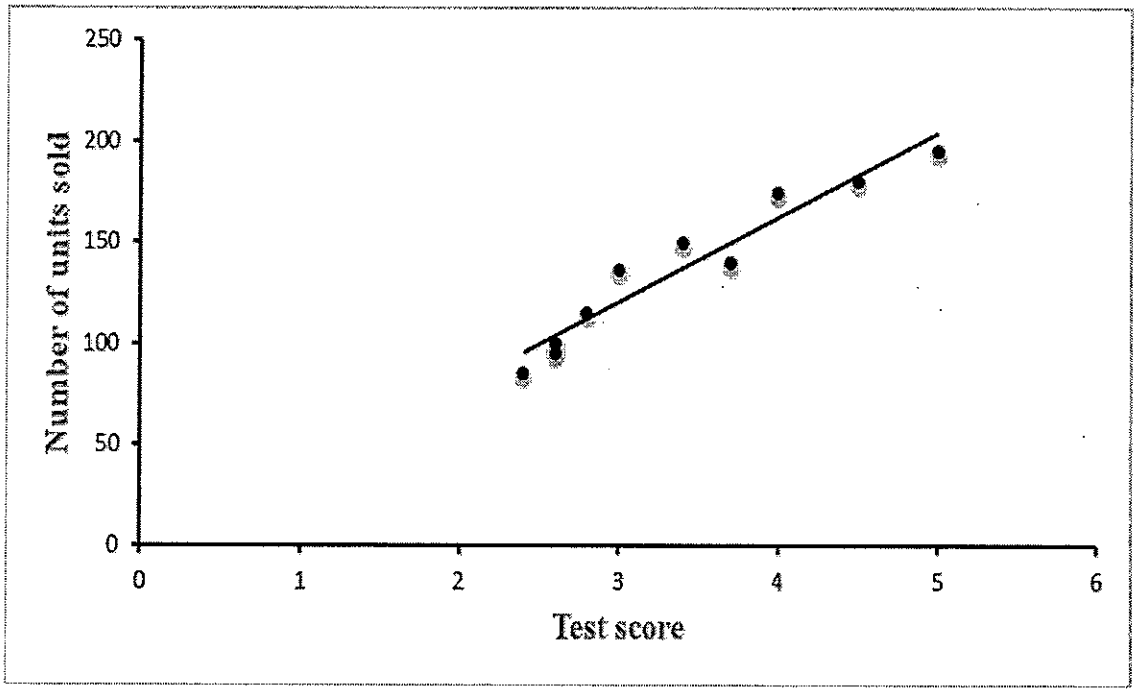
(10 marks)

Continued...

Question 3 (30 Marks)

(a) A firm administers a test on sales trainees before they go into the field. The management of the firm is interested in determining the relationship between the test scores and the sales made by the trainees. The collected data, scatterplot and summary output of the relationship between the two variables are shown below:

Test score	Number of units sold
2.6	95
3.7	140
2.4	85
4.5	180
2.6	100
5.0	195
2.8	115
3.0	136
4.0	175
3.4	150



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SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9605
R Square	0.9226
Adjusted R Square	0.9115
Standard Error	11.1246
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	10321.26	10321.26	83.4	0.00003885
Residual	8	866.2931	123.7562		
Total	9	11187.56			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.6615	15.8911	0.04163	0.96796
Test Score	40.4473	4.4291	9.13236	0.00976

(i) State the dependent variable and the independent variable for the above regression model.

(2 marks)

(ii) Write the least square regression line for the above relationship between the two variables. State the unit of measurement for each variable.

(4 marks)

(iii) What do the coefficient of the regression line tells you about the relationship between the scoring test and the sales made by the trainees?

(3 marks)

(iv) Determine the coefficient of correlation and discuss the role of this coefficient value for this model. **(4 marks)**

(v) Predict the number of units sold if the scoring test is 10. Is this estimation reliable? Explain.

(4 marks)

(vi) State the coefficient of determination and describe what it tells you.

(3 marks)

Continued...

b) An European automobile manufacturer has compiled the following information on car sales of one United States manufacturer:

Size	2005		2015	
	Price ($)	Quantity	Price ($)	Quantity
Subcompact	620.49	3200	712.54	8600
Compact	761.23	4500	833.87	7300
Sedan	904.97	46200	1060.11	38600

Compute and interpret the Laspeyres Price Index (LPI) and Paashe Price Index (PPI) for 2015 using 2005 as the base period.

(10 marks)

End of Page

A. STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \frac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\frac{\sum_{i=1}^{n} X_i^2}{n-1} - \frac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \frac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \frac{3(\bar{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution B(n, p) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \frac{X - \mu}{\sigma}$

C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\,\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) =

$\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{\hat{p}}$

Where $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \frac{(Z_{\alpha/2})^2\,\hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
One Sample Proportion Test	
$z = \frac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$	
Two Sample Mean Test	
Standard Deviation (σ) Known $z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	**Standard Deviation (σ) Not Known** $t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$
Two Sample Proportion Test	
$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \frac{X_1 + X_2}{n_1 + n_2}$	
where X_1 and X_2 are the number of successes from each population	

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	*1*	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

Where p= number of predictor

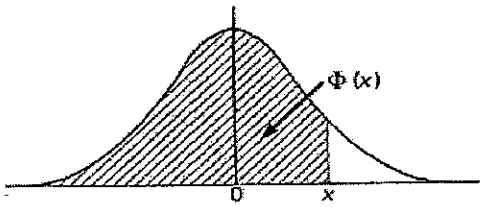
G. INDEX NUMBERS

Simple Price Index $P = \frac{p_t}{p_0} \times 100$	**Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index $P = \frac{\sum p_t}{\sum p_0}(100)$	**Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	**Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ \ Price\ Index)(Paa\ sche\ Price\ Index)}$
Paasche Price Index $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	**Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

B. STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

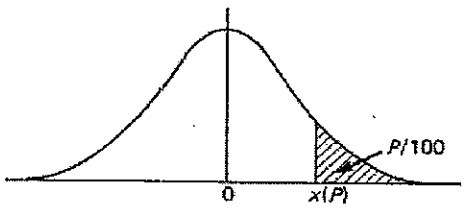
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



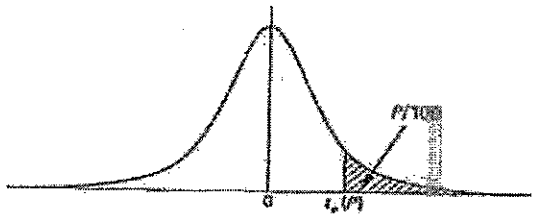
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291